

# BEHAVIOR OF REINFORCED CONCRETE SHORT RECTANGULAR COLUMNS STRENGTHENED BY STEEL LATTICE FRAMED JACKET

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**Abstract:** The complete derivation of the more exact solution that controlled the behaviour of reinforced concrete short rectangular column strengthened by steel lattice frame jacket is presented including the effect of bond slip between the concrete and reinforcement and that between concrete and steel jacket. Bond between these materials is considered well through assuming an existing of distributed frictional springs. An exact displacement field is derived utilizing the principle of minimum strain energy following Euler – Cauchy formula. Stress - strain diagram for confined concrete by internal reinforcement and external steel jacket is suggested. An empirical equation that controls the effect of interface spring stiffness between concrete and steel jacket is found through experimental investigation. A computer program is prepared using visual basic language and designated as (ARCC – SSJ)<sup>1</sup> including all the above work. The validity and efficiency of the presented finite element is tested through comparison with others experimental results. Present study shows good agreement.

## 1 INTRODUCTION

Sometimes the engineering problem may be solved through expressing it in a mathematical model. The model is often a set of differential equations which have to be solved for the behaviour of the system. A mathematical expression is introduced in the present study that controls the behaviour of reinforced concrete short rectangular columns strengthened by steel lattice framed jacket. The study presents the derivation of tangent stiffness matrix of the case mentioned previously. Several previous studies were presented by others for the same problem following either experimental investigation <sup>[1]</sup> or presenting an analytical method<sup>[2]</sup>.

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<sup>1</sup>Name of prepared program which is refer to (Analysis of Reinforced Concrete Column Strengthened by Steel Jacket).

## 2 THEORETICAL MODEL

### 2.1 Displacement field

Starting from concept of strain energy that is defined as the energy stored in a body due to axial deformation (i.e. for axial member) is equal to:

$$U = \frac{1}{2} \int_0^l \sigma(x) \epsilon(x) A(x) dx \quad (1)$$

$$\epsilon(x) = u'(x) \quad (2)$$

$$E = \frac{\sigma}{\epsilon} \quad (3)$$

$$\sigma(x) = E u'(x) \quad (4)$$

$$\pi_p = \frac{1}{2} \int_0^l u'^2(x) E A(x) dx \quad (5)$$

Utilizing the process suggested by Alwash<sup>[3]</sup>, the strain energy of axial element composed of three materials including the effect of bond slip can be written as:

$$U = \left[ \frac{1}{2} \int_0^l \sigma_c \epsilon_c A_c + \frac{1}{2} \int_0^l \sigma_{rs} \epsilon_{rs} A_{rs} + \frac{1}{2} \int_0^l \sigma_{sj} \epsilon_{sj} A_{sj} + \frac{1}{2} \int_0^l k_{scr} (u(x)_{rs} - u(x)_c)^2 + \frac{1}{2} \int_0^l k_{scj} (u(x)_{sj} - u(x)_c)^2 \right] dx \quad (6)$$

The above equation can be re-written as :

$$U = \left[ \frac{1}{2} \int_0^l u'^2 E_c A_c + \frac{1}{2} \int_0^l u'^2 E_{rs} A_{rs} + \frac{1}{2} \int_0^l u'^2 E_{sj} A_{sj} + \frac{1}{2} \int_0^l k_{scr} (u(x)_{rs} - u(x)_c)^2 + \frac{1}{2} \int_0^l k_{scj} (u(x)_{sj} - u(x)_c)^2 \right] dx \quad (7)$$

where:

$A_c, A_{rs}, A_{sj}$  = cross section area of concrete, reinforcement and steel jacket respectively.

$k_{scr}$  = interface spring stiffness between concrete and reinforcement

$k_{scj}$  = interface spring stiffness between concrete and steel jacket

Then, for minimum U, utilizing Euler – Lagrange formula:

$$\frac{\partial F}{\partial u_c} - \frac{d}{dx} \left( \frac{\partial F}{\partial u'_c} \right) = 0 \quad (8)$$

$$\frac{\partial F}{\partial u_{rs}} - \frac{d}{dx} \left( \frac{\partial F}{\partial u'_{rs}} \right) = 0 \quad (9)$$

$$\frac{\partial F}{\partial u_{sj}} - \frac{d}{dx} \left( \frac{\partial F}{\partial u'_{sj}} \right) = 0 \quad (10)$$

For  $E_c A_c = \text{constant}$ ,  $E_{rs} A_{rs} = \text{constant}$ ,  $E_{sj} A_{sj} = \text{constant}$ ,  $k_{scr} = \text{constant}$ ,  $k_{scj} = \text{constant}$  and from equations (7), (8), (9), (10), the following differential equations for the problem are obtained as :

$$u''_c + \frac{k_{scr}}{E_c A_c} (u_{rs} - u_c) + \frac{k_{scj}}{E_c A_c} (u_{sj} - u_c) = 0 \quad (11)$$

$$u''_{rs} - \frac{k_{scr}}{E_{rs}A_{rs}}(u_{rs} - u_c) = 0 \quad (12)$$

$$u''_{sj} - \frac{k_{scj}}{E_{sj}A_{sj}}(u_{sj} - u_c) = 0 \quad (13)$$

Solving the differential equations (11), (12) and (13) to find the displacement field for considered case:

$$u_c = A1 e^{\lambda_1 x} + A2 e^{-\lambda_1 x} + A3 e^{\lambda_2 x} + A4 e^{-\lambda_2 x} + A5 x + A6 \quad (14)$$

$$u_{rs} = B1 e^{\lambda_1 x} + B2 e^{-\lambda_1 x} + B3 e^{\lambda_2 x} + B4 e^{-\lambda_2 x} + B5 x + B6 \quad (15)$$

$$u_{sj} = C1 e^{\lambda_1 x} + C2 e^{-\lambda_1 x} + C3 e^{\lambda_2 x} + C4 e^{-\lambda_2 x} + C5 x + C6 \quad (16)$$

where:

$$\frac{k_{scr}}{E_c A_c} = n1$$

$$\frac{k_{scj}}{E_c A_c} = n2$$

$$\frac{k_{scr}}{E_{rs} A_{rs}} = n3$$

$$\frac{k_{scj}}{E_{sj} A_{sj}} = n4$$

$$\lambda_1 = \sqrt{\frac{N_t + \sqrt{N_t^2 - 4(n3n4 + n1n4 + n2n3)}}{2}}$$

$$\lambda_2 = \sqrt{\frac{N_t - \sqrt{N_t^2 - 4(n3n4 + n1n4 + n2n3)}}{2}}$$

## 2.2. Stiffness matrix

To find the relation between the constants ( $A1, A2, A3, \dots$ ) and ( $B1, B2, B3, \dots$ ) use equation number (12) while the relation between the constants ( $A1, A2, A3, \dots$ ) and ( $C1, C2, C3, \dots$ ) will be obtained through using equation (13)

Re-writing the displacement field in terms of constants (A) as:

$$u_c = A1 e^{\lambda_1 x} + A2 e^{-\lambda_1 x} + A3 e^{\lambda_2 x} + A4 e^{-\lambda_2 x} + A5 x + A6 \quad (17)$$

$$u_{rs} = \frac{n3 A1}{n3 - \lambda_1^2} e^{\lambda_1 x} + \frac{n3 A2}{n3 - \lambda_1^2} e^{-\lambda_1 x} + \frac{n3 A3}{n3 - \lambda_2^2} e^{\lambda_2 x} + \frac{n3 A4}{n3 - \lambda_2^2} e^{-\lambda_2 x} + A5 x + A6 \quad (18)$$

$$u_{sj} = \frac{n4 A1}{n4 - \lambda_1^2} e^{\lambda_1 x} + \frac{n4 A2}{n4 - \lambda_1^2} e^{-\lambda_1 x} + \frac{n4 A3}{n4 - \lambda_2^2} e^{\lambda_2 x} + \frac{n4 A4}{n4 - \lambda_2^2} e^{-\lambda_2 x} + A5 x + A6 \quad (19)$$

The displacement field should be re-written in terms of nodal displacements instead of the constants ( $A1, A2, A3, \dots$ )

Applying the following boundary conditions:

$$@ x = 0 \rightarrow u_c = u_{c1}, u_{rs} = u_{rs1} \text{ and } u_{sj} = u_{sj1}$$

$$@ x = L \rightarrow u_c = u_{c2}, u_{rs} = u_{rs2} \text{ and } u_{sj} = u_{sj2}$$

$$u_{c1} = A1 + A2 + A3 + A4 + A6 \quad (20)$$

$$u_{c2} = A1 e^{\lambda_1 l} + A2 e^{-\lambda_1 l} + A3 e^{\lambda_2 l} + A4 e^{-\lambda_2 l} + A5 l + A6 \quad (21)$$

$$u_{rs1} = \frac{n3}{n3 - \lambda_1^2} A1 + \frac{n3}{n3 - \lambda_1^2} A2 + \frac{n3}{n3 - \lambda_2^2} A3 + \frac{n3}{n3 - \lambda_2^2} A4 + A6 \quad (22)$$

$$u_{rs2} = \frac{n3 A1}{n3 - \lambda_1^2} e^{\lambda_1 l} + \frac{n3 A2}{n3 - \lambda_1^2} e^{-\lambda_1 l} + \frac{n3 A3}{n3 - \lambda_2^2} e^{\lambda_2 l} + \frac{n3 A4}{n3 - \lambda_2^2} e^{-\lambda_2 l} + A5 l + A6 \quad (23)$$

$$u_{sj1} = \frac{n4}{n4 - \lambda_1^2} A1 + \frac{n4}{n4 - \lambda_1^2} A2 + \frac{n4}{n4 - \lambda_2^2} A3 + \frac{n4}{n4 - \lambda_2^2} A4 + A6 \quad (24)$$

$$u_{sj2} = \frac{n4 A1}{n4 - \lambda_1^2} e^{\lambda_1 l} + \frac{n4 A2}{n4 - \lambda_1^2} e^{-\lambda_1 l} + \frac{n4 A3}{n4 - \lambda_2^2} e^{\lambda_2 l} + \frac{n4 A4}{n4 - \lambda_2^2} e^{-\lambda_2 l} + A5 l + A6 \quad (25)$$

Putting equations (20), (21), (22), (23), (24) and (25) in a matrix form:

$$\begin{pmatrix} u_{c1} \\ u_{c2} \\ u_{rs1} \\ u_{rs2} \\ u_{sj1} \\ u_{sj2} \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ e^{\lambda_1 l} & e^{-\lambda_1 l} & e^{\lambda_2 l} & e^{-\lambda_2 l} & l & 1 \\ \frac{n3}{n3 - \lambda_1^2} & \frac{n3}{n3 - \lambda_1^2} & \frac{n3}{n3 - \lambda_2^2} & \frac{n3}{n3 - \lambda_2^2} & 0 & 1 \\ \frac{n3}{n3 - \lambda_1^2} e^{\lambda_1 l} & \frac{n3}{n3 - \lambda_1^2} e^{-\lambda_1 l} & \frac{n3}{n3 - \lambda_2^2} e^{\lambda_2 l} & \frac{n3}{n3 - \lambda_2^2} e^{-\lambda_2 l} & l & 1 \\ \frac{n4}{n4 - \lambda_1^2} & \frac{n4}{n4 - \lambda_1^2} & \frac{n4}{n4 - \lambda_2^2} & \frac{n4}{n4 - \lambda_2^2} & 0 & 1 \\ \frac{n4}{n4 - \lambda_1^2} e^{\lambda_1 l} & \frac{n4}{n4 - \lambda_1^2} e^{-\lambda_1 l} & \frac{n4}{n4 - \lambda_2^2} e^{\lambda_2 l} & \frac{n4}{n4 - \lambda_2^2} e^{-\lambda_2 l} & l & 1 \end{bmatrix} \begin{pmatrix} A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \end{pmatrix} \quad (26)$$

In a symbolic form

$$\{e\}_{d.o.f.} = [R]_{6 \times 6} \{A\}_{6 \times 1} \quad (27)$$

$$\{A\}_{6 \times 1} = [R]^{-1} \{e\} = [G] \{e\} \quad (28)$$

Use the Gauss-Jordan elimination method to find the inverse of matrix  $[R]$ :

$$[R]^{-1} = \begin{bmatrix} I1 & I2 & I3 & I4 & I5 & I6 \\ I7 & I8 & I9 & I10 & I11 & I12 \\ I13 & I14 & I15 & I16 & I17 & I18 \\ I19 & I20 & I21 & I22 & I23 & I24 \\ I25 & I26 & I27 & I28 & I29 & I30 \\ I31 & I32 & I33 & I34 & I35 & I36 \end{bmatrix} \quad (29)$$

Thus, the displacement field became:

$$\begin{aligned} u_c = & (I1 e^{\lambda_1 x} + I7 e^{-\lambda_1 x} + I13 e^{\lambda_2 x} + I19 e^{-\lambda_2 x} + I25 x + I31) u_{c1} + \\ & (I2 e^{\lambda_1 x} + I8 e^{-\lambda_1 x} + I14 e^{\lambda_2 x} + I20 e^{-\lambda_2 x} + I26 x + I32) u_{c2} + \\ & (I3 e^{\lambda_1 x} + I9 e^{-\lambda_1 x} + I15 e^{\lambda_2 x} + I21 e^{-\lambda_2 x} + I27 x + I33) u_{rs1} + \\ & (I4 e^{\lambda_1 x} + I10 e^{-\lambda_1 x} + I16 e^{\lambda_2 x} + I22 e^{-\lambda_2 x} + I28 x + I34) u_{rs2} + \\ & (I5 e^{\lambda_1 x} + I11 e^{-\lambda_1 x} + I17 e^{\lambda_2 x} + I23 e^{-\lambda_2 x} + I29 x + I35) u_{sj1} + \\ & (I6 e^{\lambda_1 x} + I12 e^{-\lambda_1 x} + I18 e^{\lambda_2 x} + I24 e^{-\lambda_2 x} + I30 x + I36) u_{sj2} \end{aligned} \quad (30)$$

$$\begin{aligned}
 u_{rs} = & \left( \frac{n^3}{n^3 - \lambda_1^2} I1 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I7 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I13 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I19 e^{-\lambda_2 x} + I25 x + I31 \right) u_{c1} + \\
 & \left( \frac{n^3}{n^3 - \lambda_1^2} I2 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I8 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I14 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I20 e^{-\lambda_2 x} + I26 x + I32 \right) u_{c2} + \\
 & \left( \frac{n^3}{n^3 - \lambda_1^2} I3 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I9 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I15 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I21 e^{-\lambda_2 x} + I27 x + I33 \right) u_{rs1} + \\
 & \left( \frac{n^3}{n^3 - \lambda_1^2} I4 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I10 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I16 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I22 e^{-\lambda_2 x} + I28 x + I34 \right) u_{rs2} + \\
 & \left( \frac{n^3}{n^3 - \lambda_1^2} I5 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I11 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I17 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I23 e^{-\lambda_2 x} + I29 x + I35 \right) u_{sj1} + \\
 & \left( \frac{n^3}{n^3 - \lambda_1^2} I6 e^{\lambda_1 x} + \frac{n^3}{n^3 - \lambda_1^2} I12 e^{-\lambda_1 x} + \frac{n^3}{n^3 - \lambda_2^2} I18 e^{\lambda_2 x} + \frac{n^3}{n^3 - \lambda_2^2} I24 e^{-\lambda_2 x} + I30 x + I36 \right) u_{sj2} \quad (31)
 \end{aligned}$$

$$\begin{aligned}
 u_{sj} = & \left( \frac{n^4}{n^4 - \lambda_1^2} I1 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I7 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I13 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I19 e^{-\lambda_2 x} + I25 x + I31 \right) u_{c1} + \\
 & \left( \frac{n^4}{n^4 - \lambda_1^2} I2 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I8 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I14 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I20 e^{-\lambda_2 x} + I26 x + I32 \right) u_{c2} + \\
 & \left( \frac{n^4}{n^4 - \lambda_1^2} I3 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I9 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I15 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I21 e^{-\lambda_2 x} + I27 x + I33 \right) u_{rs1} + \\
 & \left( \frac{n^4}{n^4 - \lambda_1^2} I4 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I10 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I16 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I22 e^{-\lambda_2 x} + I28 x + I34 \right) u_{rs2} + \\
 & \left( \frac{n^4}{n^4 - \lambda_1^2} I5 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I11 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I17 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I23 e^{-\lambda_2 x} + I29 x + I35 \right) u_{sj1} + \\
 & \left( \frac{n^4}{n^4 - \lambda_1^2} I6 e^{\lambda_1 x} + \frac{n^4}{n^4 - \lambda_1^2} I12 e^{-\lambda_1 x} + \frac{n^4}{n^4 - \lambda_2^2} I18 e^{\lambda_2 x} + \frac{n^4}{n^4 - \lambda_2^2} I24 e^{-\lambda_2 x} + I30 x + I36 \right) u_{sj2} \quad (32)
 \end{aligned}$$

The strain – displacement relation is given in the following expressions:

$$\epsilon_c = \frac{du_c}{dx} \quad (33)$$

$$\epsilon_{sr} = \frac{du_{sr}}{dx} \quad (34)$$

$$\epsilon_{sj} = \frac{du_{sj}}{dx} \quad (35)$$

Then, in a matrix form:

$$\begin{Bmatrix} \epsilon_c \\ \epsilon_{sr} \\ \epsilon_{sj} \end{Bmatrix} = \begin{bmatrix} Q1 & Q2 & Q3 & Q4 & Q5 & Q6 \\ Q7 & Q8 & Q9 & Q10 & Q11 & Q12 \\ Q13 & Q14 & Q15 & Q16 & Q17 & Q18 \end{bmatrix} \begin{Bmatrix} u_{c1} \\ u_{c2} \\ u_{rs1} \\ u_{rs2} \\ u_{sj1} \\ u_{sj2} \end{Bmatrix} \quad (36)$$

Or, in a symbolic form

$$\{\epsilon\} = [B]\{u\} \quad (37)$$

where

$$[B] = \begin{bmatrix} Q1 & Q2 & Q3 & Q4 & Q5 & Q6 \\ Q7 & Q8 & Q9 & Q10 & Q11 & Q12 \\ Q13 & Q14 & Q15 & Q16 & Q17 & Q18 \end{bmatrix}$$

$$Q1 = \lambda_1 I1 e^{\lambda_1 x} - \lambda_1 I7 e^{-\lambda_1 x} + \lambda_2 I13 e^{\lambda_2 x} - \lambda_2 I19 e^{-\lambda_2 x} + I25$$

$$Q2 = \lambda_1 I2 e^{\lambda_1 x} - \lambda_1 I8 e^{-\lambda_1 x} + \lambda_2 I14 e^{\lambda_2 x} - \lambda_2 I20 e^{-\lambda_2 x} + I26$$

$$Q3 = \lambda_1 I3 e^{\lambda_1 x} - \lambda_1 I9 e^{-\lambda_1 x} + \lambda_2 I15 e^{\lambda_2 x} - \lambda_2 I21 e^{-\lambda_2 x} + I27$$

$$Q4 = \lambda_1 I4 e^{\lambda_1 x} - \lambda_1 I10 e^{-\lambda_1 x} + \lambda_2 I16 e^{\lambda_2 x} - \lambda_2 I22 e^{-\lambda_2 x} + I28$$

$$Q5 = \lambda_1 I5 e^{\lambda_1 x} - \lambda_1 I11 e^{-\lambda_1 x} + \lambda_2 I17 e^{\lambda_2 x} - \lambda_2 I23 e^{-\lambda_2 x} + I29$$

$$Q6 = \lambda_1 I6 e^{\lambda_1 x} - \lambda_1 I12 e^{-\lambda_1 x} + \lambda_2 I18 e^{\lambda_2 x} - \lambda_2 I24 e^{-\lambda_2 x} + I30$$

$$Q7 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I1 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I7 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I13 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I19 e^{-\lambda_2 x} + I25$$

$$Q8 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I2 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I8 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I14 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I20 e^{-\lambda_2 x} + I26$$

$$Q9 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I3 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I9 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I15 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I21 e^{-\lambda_2 x} + I27$$

$$Q10 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I4 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I10 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I16 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I22 e^{-\lambda_2 x} + I28$$

$$Q11 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I5 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I11 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I17 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I23 e^{-\lambda_2 x} + I29$$

$$Q12 = \frac{n3\lambda_1}{n3 - \lambda_1^2} I6 e^{\lambda_1 x} - \frac{n3\lambda_1}{n3 - \lambda_1^2} I12 e^{-\lambda_1 x} + \frac{n3\lambda_2}{n3 - \lambda_2^2} I18 e^{\lambda_2 x} - \frac{n3\lambda_2}{n3 - \lambda_2^2} I24 e^{-\lambda_2 x} + I30$$

$$Q13 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I1 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I7 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I13 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I19 e^{-\lambda_2 x} + I25$$

$$Q14 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I2 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I8 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I14 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I20 e^{-\lambda_2 x} + I26$$

$$Q15 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I3 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I9 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I15 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I21 e^{-\lambda_2 x} + I27$$

$$Q16 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I4 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I10 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I16 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I22 e^{-\lambda_2 x} + I28$$

$$Q17 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I5 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I11 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I17 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I23 e^{-\lambda_2 x} + I29$$

$$Q18 = \frac{n4\lambda_1}{n4 - \lambda_1^2} I6 e^{\lambda_1 x} - \frac{n4\lambda_1}{n4 - \lambda_1^2} I12 e^{-\lambda_1 x} + \frac{n4\lambda_2}{n4 - \lambda_2^2} I18 e^{\lambda_2 x} - \frac{n4\lambda_2}{n4 - \lambda_2^2} I24 e^{-\lambda_2 x} + I30$$

The stiffness matrix will be in an integration form as:

$$[K]_{6 \times 6} = \int_0^L [B]_{6 \times 3}^T [E]_{3 \times 3} [B]_{3 \times 6} dx \quad (38)$$

where

$$[E]_{3 \times 3} = \begin{bmatrix} E_c & 0 & 0 \\ 0 & E_{rs} & 0 \\ 0 & 0 & E_{sj} \end{bmatrix}$$

A numerical integration was carried out, using 3- point Gaussian Quadrature rule, to evaluate the stiffness matrix.

## 2.3 Proposed stress – strain diagram for confined concrete:

The proposed stress – strain diagram for confined concrete will be achieved by modifying a stress – strain diagram for unconfined concrete.

At a pointed value of strain, the corresponding stress value will be larger in confined concrete, the ultimate strength of confined concrete will be taken from Eurocode 8 provision and the effect of confining pressure will be based on Campione<sup>[2]</sup> equations.

### 2.3.1 Stress – strain diagram for unconfined concrete

The proposed compressive uniaxial stress-strain relationship for the concrete model was obtained using the following equations to complete the multilinear isotropic stress-strain curve for the concrete.

$$\sigma = \frac{E_c \varepsilon}{1 + (\frac{\varepsilon}{\varepsilon_o})^2} \quad \varepsilon_1 < \varepsilon < \varepsilon_o \quad (39)$$

$$\sigma = f'_c \quad \varepsilon \geq \varepsilon_o \quad (40)$$

$$\varepsilon_o = \frac{2 f'_c}{E_c} \quad (41)$$

$$\sigma = E_c \cdot \varepsilon \quad \varepsilon < \varepsilon_1 \quad (42)$$

$$\varepsilon_1 = \frac{0.3 f'_c}{E_c} \quad (43)$$

where

$\sigma$  = stress at any strain, MPa

$\varepsilon$  = strain at stress  $\sigma$

$\varepsilon_o$  = strain at the ultimate compressive strength ( $f'_c$ )

The multilinear isotropic stress-strain relationship requires the first point of the curve to be satisfying Hooke's law. The simplified stress-strain curve for concrete model is constructed from six points connected by straight lines. The curve starts at zero stress and strain. Point number 1, at  $0.30 f'_c$ , is calculated for the stress-strain relationship of the concrete in the linear range from equation (42). Point of numbers 2, 3, and 4 are obtained from equation (39), in which ( $\varepsilon_o$ ) is calculated from equation (41). Point of number 5 is at ( $\varepsilon_o$ ) and ( $f'_c$ ). In this study, an assumption was made of perfectly plastic behaviour after point number 5.

### 2.3.2 Eurocode provisions for compressive strength in confined concrete

The strength of confined concrete may be evaluated from:

$$f_{cc} = f'_c \left[ 1 + 3.7 \left( \frac{C_p}{f'_c} \right)^{0.87} \right] \quad (44)$$

Equation (44) will be adopted to express the ultimate strength of confined concrete. The strain  $\varepsilon_{cc}$  is evaluated according to Mander et al. [4] as:

$$\varepsilon_{cc} = \varepsilon_o \left[ 1 + 5 \left( \frac{f_{cc}}{f'_c} - 1 \right) \right] \quad (45)$$

The simplified stress-strain curve for confined concrete model is also constructed from six points connected by straight lines with an assumption was made of perfectly plastic behaviour after point number (5) as shown in figure (1).

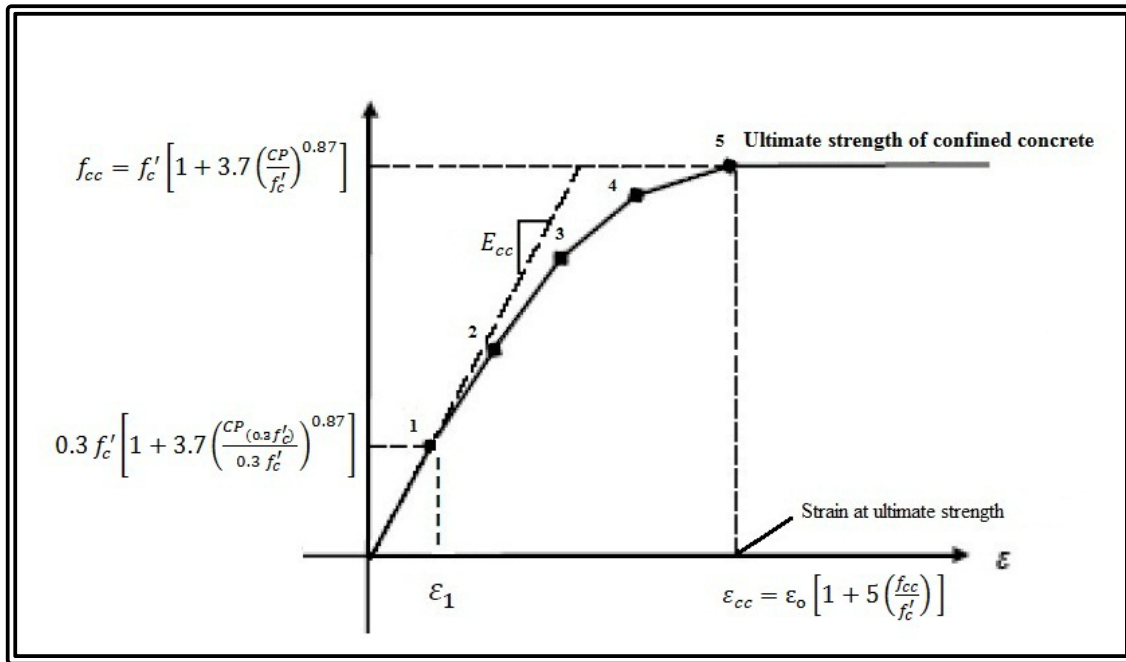


Figure (1): Proposed Stress – Strain Diagram for Confined Concrete

### 2.3.3 Confining pressure

Confining pressure (Cp) due to steel jacket is calculated adopting the model of Campione [2]:

$$C_{p \text{ steel jacket}} = 1.33 f_{yb} e^{-\frac{1.5s}{b}} s^{-1} \left( \frac{Ls}{s t_a} + \frac{L2}{s_2 t_b} \right)^{-1} \quad (46)$$

where

$f_{yb}$  = yield stress of battens, MPa



$S$  = spacing between two successive battens, mm  
 $L_s$  = angle side length, mm  
 $t_a$  = thickness of steel angles, mm  
 $t_b$  = thickness of steel battens, mm  
 $s_2$  = width of battens, mm  
 $b$  = column width, mm  
 $L_2 = (b - 2L_s) / 2$ , mm

Confining pressures due to internal longitudinal bars and stirrups (by Campione<sup>[2]</sup>) will be:

$$C_{p_{\text{reinforcement}}} = \frac{2\pi f_y \varphi_{st}^2}{4 s_{st} b} + n_b \frac{384}{3 s_{st}^4} E_r \frac{\pi \varphi_l^3}{64} * \frac{b}{2} (v \varepsilon_{sp} - \varepsilon_{yst}) \quad (47)$$

where

$\varphi_{st}$  = diameter of transverse stirrups, mm  
 $\varphi_l$  = diameter of longitudinal reinforcement, mm  
 $s_{st}$  = spacing between two successive stirrups, mm  
 $n_b$  = the number of side bars  
 $E_{rs}$  = modulus of elasticity for reinforcement, MPa  
 $v$  = Poisson's ratio  
 $\varepsilon_{sp}$  = the strain corresponding to concrete cover spalling  
 $\varepsilon_{yst}$  = yield strain of the stirrups

The pressure that has obtained from equation (46) in addition to that determined from equation (47) are presenting the total confining for the system:

$$C_p = C_{p_{\text{steel jacket}}} + C_{p_{\text{reinforcement}}} \quad (48)$$

### 3 INTERFACE SPRING BETWEEN STEEL JACKET AND CONCRETE

The bond strength between a rebar and the surrounding concrete is generally due to three components:

1. Chemical adhesion;
2. Friction;
3. Mechanical interlock between reinforcement and concrete.

The interface spring stiffness between the steel jacket and concrete do not include the chemical adhesion nor mechanical interlock also there is no available data to estimate it so this was found experimentally.

Another reason that leads to do this experimentally is that the parameters of any formula which is used to evaluate the stiffness of the interface spring between the concrete and steel jacket do not take the additional bond which is from the effect of transverse expansion of the axially – loaded column due to Poisson's ratio.

The suggested empirical equation of the interface spring stiffness between concrete and steel jacket becomes

$$K_{sc} = \frac{1.83}{L_{sj}} (E_s + E_c) * L_s * C_{p_{\text{steel jacket}}} \quad (49)$$

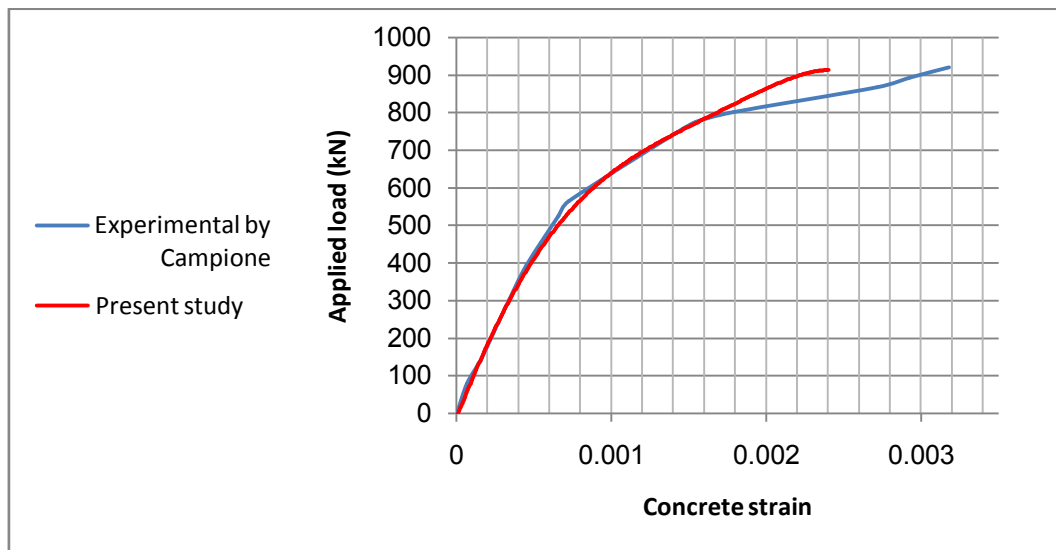
#### 4 APPLICATIONS

Case (1): Comparison between Campione experimental results with those of the present study. The dimensions and properties of column specimens tested by Campione are summarized in Table (1).

**Table (1):** Details of Campione Specimen

Column		Longitudinal bar		Stirrups		Angles		Strips	
Cross section dimension mm	215*215	diam. mm	12	diam. mm	6	Size mm	30	Size mm	30
length mm	1200	No. of bar	4	Spacing mm	200	Thickness mm	3	Thickness mm	3
Fc' MPa	10	yield stress MPa	461	yield stress MPa	400	yield stress MPa	239	Spacing mm	200
								yield stress MPa	239

The results of Campione experimental work and present study are shown in figure (2):



**Figure (2):** Comparison Between The Experimental Results (Campione ) and The Present Study (Loaded Angles)

Case (2): Load carrying capacity with corresponding failure state when eccentric load is applied, three cases has been taken into consideration. Those were tested by G. Roca et. al. (2010) which have the same details and different eccentricities as shown in Table (2).

**Table (2):** Details of Garzon – Roca et al. (2010)<sup>[2]</sup> Specimen

Column		Longitudinal bar		Stirrups		Angles		Strips	
Cross section dimension mm	260*260	diam. mm	12	diam. mm	6	Size mm	60	Size mm	140
length mm	1200	No. of bar	4	Spacing mm	200	Thickness mm	6	Thickness mm	8
Fc' MPa	12	yield stress MPa	500	yield stress MPa	500	yield stress MPa	275	Spacing mm	182
								yield stress MPa	275

The results of experimental test by G. Roca and present study were briefed in Table (3).

**Table (3):** Experimental Results (Garzon-Roca et al. 2010)<sup>[2]</sup> and Present Study (Loaded Angles)

Eccentricity mm	Experimental Axial load capacity kN	Present study Axial load capacity kN	Experimental Moment capacity kN.m	Present study Moment capacity kN.m
0	2481.13	2465	0	0
155	1000	995	154.4	154.2
615	349	342	214.4	210.3

## 5 CONCLUSIONS

The present study introduced a new simple and efficient element which can understand the behaviour of reinforced concrete short rectangular columns strengthened by steel lattice

framed jacket. New model that controls the interface spring stiffness between concrete and steel jacket including the effect of transverse expansion of the axially – loaded column due to Poisson's ratio. The model can be used to study the interface nonlinearity between concrete and steel jacket which is not considered before.

## REFERENCES

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